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Analysis of Stress Concentration Area about the Brace of the Concrete Wall at Early Age

Antanas Žiliukas[1], Giedrius Žiogas[1*]

[1]*Kaunas University of Technology, Strength and Fracture Mechanics Centre, Kęstučio st. 27, LT-44025 Kaunas, Lithuania*

** corresponding author: ziogas.giedrius@gmail.com*

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Scientists recently focus on concrete's hardening early age and its influence to solidity of a structure. Because of complex physical – chemical processes and developed strains, stresses appear in concrete and after they exceed tensile strength of concrete - develops cracks. In practice it is noted that a structure often cracks prior to commencement of exploitation. Therefore this article analyzes the influence of stresses caused by autogenous shrinkage over solidity of structure. The importance of stresses caused by concrete shrinkage significantly increases in places where a cross-section shifts. The stress concentration area develops at these points. One of the stress concentration areas is around the formwork's transverse brace and stresses due to autogeneous shrinkage are solved. To define stresses, analytical and finite element methods are used. The stresses concentration area is calculated more precisely using the finite elements method, the results obtained are exhaustive and it allows to get a clearer picture of stresses.

Keywords: *Autogenous shrinkage, cracks, early age, stress concentration area, finite element method.*

1. Introduction

In recent years, there have been a number of scientific works where the influence of concrete early age hardening upon development of cracks is researched. While concrete hardens there are complex chemical and physical processes under proceeding, if not controlled they can negatively impact a structure.

When cracks appear in reinforced concrete structures, the structure loses solidity, bearing capacity and it has negative impact on exploitation. So it is important to assess impact from concrete shrinkage and to suppose possible areas of stress concentration. A concentration area develops in places where cross-section of structure changes, i. e. decrease (Viau 2010). Therefore, it is important to know these weak points and to solve the problem as per technological and structural aspect reducing the impact of stresses upon solidity of structure.

One of those places is around transverse brace of formworks, where structure is weaken by transverse continuous opening. It is known from practice that cracks are often noticed at these places. It should be noted that similar problems, when the stress concentration fields develop, are researched by scientists (Rees et al. 2012, Luo et al. 2012)The peculiarities of monolithic concrete pouring, factors determining the development of cracks as well as prevention methods were partly discussed in the publications analyzed (Žiogas and Jočiūnas 2007). Similar cases where concentration fields of cracks develop are investigated by scientists (Rees et al. 2012, Luo *et al.* 2012), who indicate the appearance of cracks because of stress concentration as negative impact on the exploitation of structure. Scientific works establish that depending on attenuation of cross-section, stresses can increase from 3 to 4 times.

In practice it is noted that a structure often cracks prior to commencement of exploitation. During the hydration a volume of concrete changes and where areas are restricted because of the concrete shrinkage, inward stresses appear in concrete and after they exceed concrete tensile strength cracks appear (Hansen 2011). The factors that influence shrinkage of concrete are given in the picture 1 (Holt and Leivo 2004).

As mentioned above, after appearance of concrete strains, tensile stresses appear as well $\sigma_t = E_c \cdot \varepsilon_c$. (here: E_c – modulus of elasticity of concrete; ε_c – strain of concrete). When tensile stresses appeared because of strains exceed concrete tensile strength $\sigma_t > f_{ctm}$ cracks occur.

This article analyzes the development of concentration area around transverse brace of formwork when autogenous shrinkage strains are present, shrinkage conditional strain is not estimated because the structure are restricted with formworks that prevent moisture loses from the structure.

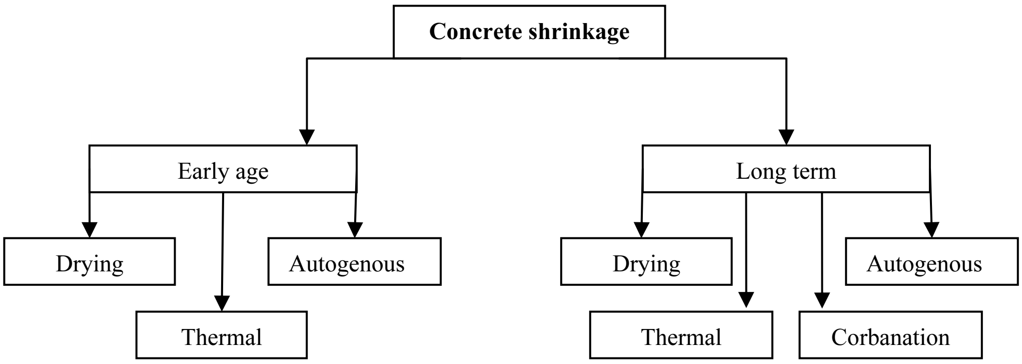

Fig. 1. *Diagram of shrinkage stages and types*

2. Methods

Calculation methods of concrete strain-stress at early age

One of the processes causing cracks in concrete at the early stage of hardening is total shrinkage which develops while concrete is hardening.

Total shrinkage strain consists of two components (Eurocode 2, 2005):

- drying shrinkage strain;
- autogenous shrinkage strain

Total shrinkage strain values depend on the composition of the mix, water-cement ratio, time of hardening, geometrical characteristics and the surroundings (relative humidity of the air).

Since the structure is restrained by formwork, concrete strain caused by moisture loss is not considered,but the autogenous shrinkage is proceeded. Peak values of total shrinkage strain will be found along the xx direction of the wall. In this case stresses due to the developed strains can be calculated in the following manner:

$$\sigma_{xx} = \varepsilon_{xx} \cdot E_{c(t)} \quad (1)$$

Where: ε_{xx} – autogenous shrinkage, $E_{c(t)}$ – modulus of elasticity at age t.

To determine autogenous shrinkage according to the following mathematical model (JCI Technical Committee, Tazawa and Miyazawa, 2002):

$$\varepsilon_{c(t)} = \gamma \cdot \varepsilon_{c0} \cdot (W/C) \cdot \beta_t \quad (2)$$

when $0{,}2 \leq W/C \leq 0{,}5$:

$$\varepsilon_{c0}(W/C) = 3070 \exp[-7{,}2(W/C)] \quad (3)$$

when $W/C > 0{,}5$:

$$\varepsilon_{c0}(W/C) = 80 \quad (4)$$

$$\beta_t = 1 - \exp[-a(t-t_0)^b] \quad (5)$$

here $\varepsilon_{c(t)} \cdot 10^{-6}$ is the autogenous shrinkage of concrete at age t; γ is the coefficient which assesses the variety of cement, $\gamma=1$, when regular Portland cement is used; $\varepsilon_{c0} \cdot 10^{-6}$ are the highest autogenous shrinkage strains of the cement stone with the corresponding ratio of water and binding material W/C; β_t is a coefficient which assesses autogenous shrinkage in relation to time; W/C is the water- cement ratio; t is age of concrete in days; t_0 is the initial time of binding in days; a and b are the coefficients taken from table 1.

Autogenous shrinkage of concrete (cement C=350 kg/m^3,W/C=0,415; concentration of coarse aggregate – φ_{st}= 0,375) was calculated using these dependencies.

Table 1. *Values of coefficients a and b*

W/C \ Coeff.	0,2	0,3	0,4	0,5	0,6
a	1,2	1,5	0,6	0,1	0,03
b	0,4	0,4	0,5	0,7	0,8

Fig. 2 shows the dependencies of concrete stresses due to its autogenous shrinkage and hardening time. These dependencies were obtained using formulas 1 and 5.

Compressive strength of hardening concrete were obtained by means of an industrial experiment (Žiogas et al. 2007), while its tensile strength and modulus of elasticity were calculated using the corresponding formulas and EC2 regulations. Modulus of elasticity of concrete are given in table 2.

Table 2. *Modulus of elasticity of concrete*

Hardening time, days	1	2	3	7	14	28
E_c, MPa	23920	26870	28290	30620	31990	33000

When values of internal stresses are known, the area of stress concentration around the hole can be calculated. Stresses are calculated by applying analytical and numerical methods. The former method of calculating the area of stresses concentration uses the recommended formulas (Žiliukas et al. 2010).

Radial normal stresses around hole σ_{rr} are obtained from the formula given below:

$$\sigma_{rr} = \frac{\sigma_{xx}}{2}\left(1-\frac{a^2}{r^2}\right) - \frac{\sigma_{xx}}{2}\cos 2\theta\left(1-\frac{a^2}{r^2}\right)\cdot\left(1-\frac{3a^2}{r^2}\right) \quad (6)$$

here: a is the radius of the hole 10 mm, r is the radius from the centre of the hole to any other point.

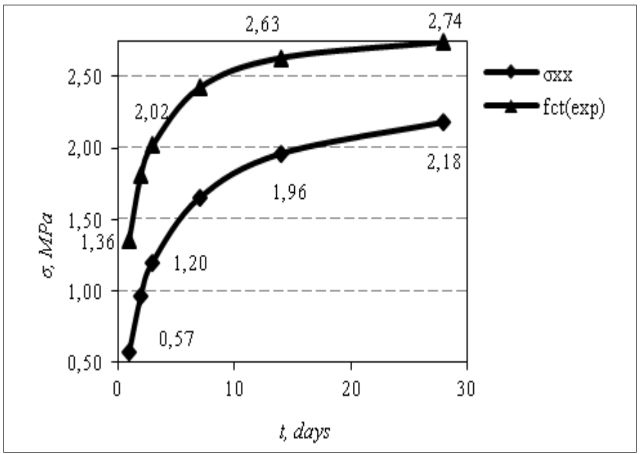

Fig. 2. *Stresses developed during the hardening of concrete and tensile strength of concrete: calculated and experimental. Here : σ_{xx} are internal stresses in concrete caused by autogenous shrinkage; $f_{ct(exp)}$ is the tensile strength of concrete calculated from experimental compressive strength*

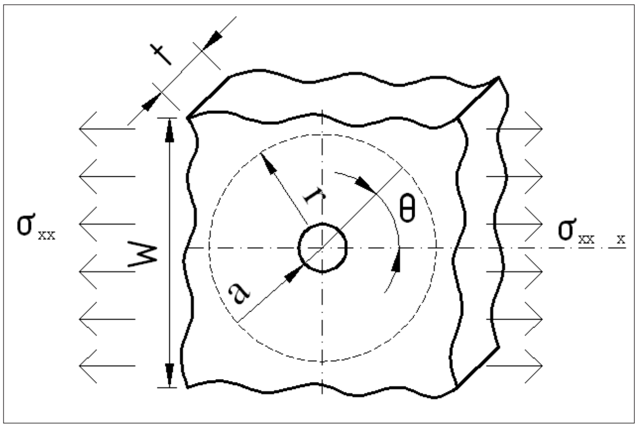

Fig. 3. *Scheme for calculating around the transverse brace of formwork. Here : σ_{xx} are the stresses due to autogenous shrinkage of concrete; a is the radius of the hole ; r is a point removed a certain distance from the centre of the hole ; θ is the angle from axis x; t is the thickness of the member; W – is the width of the element*

Stresses near the hole are calculated, therefore the radius a=r, and stresses σ_{rr}=0.

When r>>a, stresses are calculated according to the formula given below:

$$\sigma_{rr} = \frac{1}{2}\sigma_{xx}\left[1+\cos(2\theta)\right] \quad (7)$$

It can be seen from the formula that stresses σ_{rr} depend on angle θ. When θ=0, $\sigma_{rr} \approx \sigma_{xx}$, when θ =90, $\sigma_{rr} \approx 0$.

Circular normal stresses are calculated from the formula below:

$$\sigma_{\theta\theta} = \frac{\sigma_{xx}}{2}\left(1+\frac{a^2}{r^2}\right)+\frac{\sigma_{xx}}{2}\cos 2\theta\left(1+3\frac{a^4}{r^4}\right) \quad (8)$$

When a=r, stresses $\sigma_{\theta\theta}$ are calculated from:

$$\sigma_{\theta\theta} = \frac{\sigma_{xx}}{2}\left(2-4\cos(2\theta)\right) \quad (9)$$

The stress concentration, i. e. $\sigma_{\theta\theta}=3\sigma_{xx}$ is formed when angle θ=90°.

Moving further from the centre of the hole, i. e., when r>>a, stresses change with angle θ, $\sigma_{\theta\theta(0)}=0$ ir $\sigma_{\theta\theta(90)}=\sigma_{xx}$.

Tangential stresses are obtained from the following formula:

$$\sigma_{r\theta} = -\frac{\sigma_{xx}}{2}\sin 2\theta\left(1-\frac{a^2}{r^2}\right)\left(1+3\frac{a^2}{r^2}\right) \quad (10)$$

When a=r, stresses $\sigma_{r\theta}$ are calculated:

$$\sigma_{r\theta} = 0 \quad (11)$$

When angle θ=0, $\sigma_{r\theta}$ =0, circular stresses $\sigma_{\theta\theta}$ are key stresses σ_1, and radial stresses σ_{rr} are key stresses σ_2.

When a biaxial stress state is present, equivalent stresses are calculated according to Mises (Liu, 2005):

$$\sigma_i = \sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2} \quad (12)$$

3. Results and Discussion

Stresses obtained analytically are presented in table 3. While calculating the area of stress concentration, the increment of stresses due to the decreased cross – section area A_{net}=A-2r is taken into consideration (nominal stresses).

Table 3. *Stresses around the hole calculated*

Hardening time, days	Stresses, MPa			
	σ_{nom}	σ_1	σ_2	$\sigma_{i,a}$
1	0,62	1,87	0,62	**1,65**
2	1,05	3,16	1,05	**2,79**
3	1,31	3,92	1,31	**3,46**
7	1,65	4,95	1,63	**4,95**
14	2,15	6,44	2,15	**5,68**
28	2,38	7,14	2,38	**6,30**

In the numerical calculation of stresses, the finite element method uses the Ansys 12 program; the results obtained are presented in table 4. In the finite element method calculation, a geometrical model is made for ¼ of the structural member, and it is indicated in the program that the member is symmetrical around the x and y axes. Such a model does not impact calculation results; besides, fewer computer resources are used.

Table 4. *Stresses calculated using the finite element method (with Ansys 12 program)*

Hardening time Days	Stresses, MPa				Margin
	σ_{xx}	σ_1	σ_2	$\sigma_{i,s}$	$(\sigma_{i,s}-\sigma_{i,a})/\sigma_{i,s}\cdot 100\%$
1	0,57	1,71	0,56	**1,71**	3,51
2	0,96	2,88	0,95	**2,88**	3,13
3	1,19	3,57	1,18	**3,57**	3,08
7	1,65	4,95	1,63	**4,95**	0,00
14	1,96	5,87	1,94	**5,88**	3,40
28	2,18	6,47	2,11	**6,46**	2,48

Maximum margin is 3,5 % in the analytically and numerically calculated equivalent stresses σ_i.

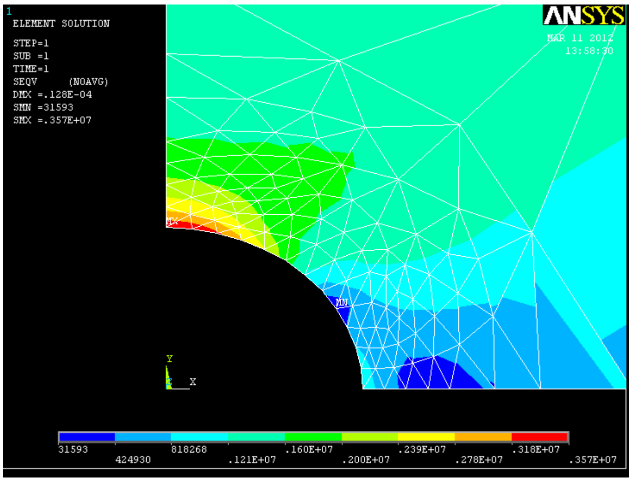

Fig. 4. *Distribution of the area of stress concentration (N/mm²) around the hole (after 3 days of hardening)*

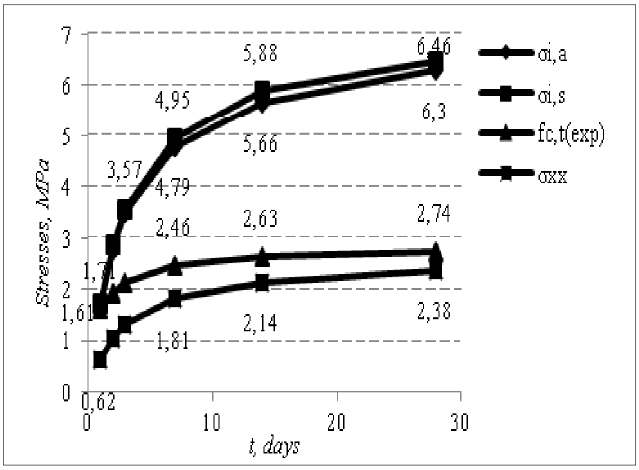

Fig. 5. *Comparison of stresses and tensile strengths of concrete. Here: σ_{xx} are stresses caused by autogenous shrinkage of concrete; $f_{ct(exp)}$ is the tensile strength of concrete calculated from experimental compressive strength; $\sigma_{i,a}$ are the stresses obtained using the analitical method; $\sigma_{i,s}$ are the stresses obtained using the numerical method with program Ansys*

The numerical problem solution allows to calculate the stress distribution around the transverse braces at any angle θ and the radius of the hole. Numerical method obtained the stress distribution throughout the structure, which allows a better analysis of the construction work and predict crack growth. Numerical method is more accurate and comprehensive as analytical method.

It is obvious from the results obtained, that stresses three times as big as those impacting the member σ_{xx} develop near the hole; they exceed the tensile strength of concrete within the first days of hardening. Micro cracks appear at these locations even before the exploitation of the structural member begins.

After the removal of the formwork, total shrinkage strain due to drying starts developing; its limit value depends on the characteristics of the concrete structure and the surrounding environment. Later, both the total shrinkage and stresses round the hole continue increasing, accompanied by the growth of the crack which can be noticed with the naked eye after the removal of the formwork .

4. Conclusions

1. An area of stress concentration round the transverse bracing of the formwork, there the value of equivalent stresses is three times as big as the acting stresses appearing due to autogenous strains of concrete. The numerical problem solution allows calculate the stress distribution around the transverse braces at any angle θ and the radius of the hole.
2. The equivalent stresses exceed concrete's tensile strength during the early days of concrete hardening and cause the opening of a crack.
3. To improve the quality of monolithic reinforced concrete structures as well as the reliability of exploitation, it is necessary to assess all the factors that influence strain-stresses behavior: when concrete mix is poured, when it hardens inside the formwork, when the formwork is removed and during the subsequent stages of hardening at surrounding environment.

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Antanas ŽILIUKAS – Kaunas University of Technology, Strength and Fracture Mechanics Centre.
Main research area: Antanas Žiliukas at the Strength and Fracture Mechanics Centre, Kaunas University of Technology. From 2000 A. Žiliukas has been a director of Strength and Fracture Mechanics Centre. He has taken part in international projects related to the design of hydrogen aircraft engines and safety systems for nuclear power plants. The author of more than 20 books, and published over 100 articles. Research interest materials mechanics, theory of elasticity and plasticity, fracture mechanics, theory of reliability to students at all levels.
Address: Kęstučio st. 27, LT-44025 Kaunas, Lithuania.
Tel.: 8-37-300431
E-mail: *antanas.ziliukas@ktu.lt*

Giedrius ŽIOGAS – affiliation Kaunas University of Technology, Strength and Fracture Mechanics Centre.
Main research area: Person maintaining a doctor’s thesis. MSc (2009, Civil engineer). Strength and fracture Mechanics Centre at Kaunas University of Technology. Research interests: fracture mechanics of reinforced concrete structures.
Address: Kęstučio st. 27, LT-44025 Kaunas, Lithuania.
Tel.: +370 605 31996
E-mail: *ziogas.giedrius@gmail.com*